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Conventional Nuclear Physics Solution of the Solar Neutrino Problem

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A basic and inherently simple alternative explanation of the solar neutrino problem is proposed based upon conventional nuclear physics. Our results for the tunneling factor, astrophysical S-factor, and our resolution are compared with rather speculative solutions commonly attempted by accepting the customary ingredients of the standard solar model. We present a more realistic solution of nuclear Coulomb barrier tunneling together with a more precise parametric representation of the astrophysical function S. We determine S from high-energy (>100 keV) $^7Be(p, \gamma)^8B$ experimental cross-section data using the new tunneling factor. This leads to a low-energy fusion cross section that is lower than previous estimates by ~26–36%, decreasing the anticipated neutrino flux close to experimentally detected values. This may resolve the missing solar neutrino flux problem.

1. INTRODUCTION

The experimental results from 1968 to 1986 from the ^{37}Cl neutrino detector (the world's only solar neutrino detector in that period) in the Homestake Mine (Davis, 1986) initiated one of the most puzzling and long-lasting problems of modern physics, known as the missing solar neutrino flux problem, or more simply the solar neutrino (ν) problem. Of the many experiments that have been conducted, the experimental neutrino deficit is a factor of 2–3 times lower than the accepted prediction from the standard solar model (SSM), as thoroughly discussed with ample references in excellent review papers (Bahcall and Ulrich, 1988; Bahcall *et al.*, 1988).

The SSM has been successful in relating the mass and composition of the sun to its luminosity and lifetime. The SSM has also been widely accepted, as it appears to be based upon well-understood nuclear physics.

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However, as we will show, this has included approximations that are inconclusively established both for high energies and for the solar energy regime. In fact the SSM has appeared to work so well that the preponderance of attempted theoretical solutions have been directed at the neutrinos, rather than the SSM.

Dating back to 1969 (Bahcall and Ulrich, 1988; Kuo and Pantaleone, 1989) it was suggested that there may be an oscillation from one neutrino family to another, i.e., ν_e oscillates to ν_μ or ν_τ during their 93 million-mile vacuum journey from the sun to the earth. Mikheyev and Smirnov (1985, 1986), based upon earlier work by Wolfenstein (1978), proposed that if neutrinos have a small rest mass, this oscillation would have large consequences near the center of the sun. The father of stellar fusion theory, Bethe, took this conjecture seriously (Bethe, 1986), and even corrected an error in the earlier calculations. Wolfenstein had proposed that a ν responds as if its mass has been increased proportional to the density of the surrounding matter. Although this would be a negligible effect for ν_μ and ν_τ, it could be a big effect for ν_e deep in the sun, where the density exceeds 130 g/cm^3. In this domain the mass of a ν_e could be greater than a ν_μ, and it could become a ν_μ as it moves out of the sun to regions of low density, making it undetectable in the Homestake detector. The much newer ^{71}Ga detectors in the Gran Sassa Laboratory in Italy and for the Soviet–American Gallium Experiment (SAGE) at Baksan in the former Soviet Union have better detection efficiency.

Although the new detectors are lessening the deficit, it has not disappeared. A crucial test of neutrino family oscillation would be that the same reaction measured at the same energy gives different results at different distances. This has yet to be demonstrated. In addition, no generally accepted measurement of neutrino masses has yet been made. Bahcall and Glashow (1987), assuming the SSM, have used the neutrino detection data at the Kamikande detector in Japan to put an upper limit on the mass of the electron neutrino at less than 11 eV. Although reports of an anomalously high neutrino mass of 17 keV first observed in 1985 (Simpson and Hime, 1989) with solid-state detectors persist, astrophysical arguments imply an upper limit of 80 eV. Extensions of the Glashow–Weinberg–Salam electroweak theory suggest the possibility of a neutrino mass between 10^{-6} and 1 eV.

A possible anomalous magnetic moment for the neutrino has also been invoked to solve the solar neutrino problem. The three neutrino families are all expected to have left-handed (negative) chirality with their spin vector always antiparallel to their momentum vector, i.e., their spin should always be counterclockwise around their momentum vector. Positive-chirality neutrinos would not be allowed by conventional theories because they would have no interactions with ordinary matter. They could neither be produced nor detected. However, a negative-chirality ν_e could have its spin flipped to positive chirality in passing through magnetic fields ~kG on the sun (Cisneros, 1971) and not be detectable at any neutrino detector. This solution might also account for an apparent drop in the neutrino flux when sunspot activity increases (larger magnetic fields) and rises when this activity decreases (Davis, 1986; Lande, 1990). A magnetic moment of 10^{-11}–10^{-10} Bohr magnetons for the ν_e would be needed for this solution to work. This is considered unlikely from a theoretical point of view (Fujikawa and Shrock, 1980). In addition, a number of scientists feel that there is not enough statistical evidence for the apparent anticorrelation of the solar neutrino flux with the sunspot cycle.

One imaginative solution (first noted, but unpublished at the time, by J. Faulkner and R. J. Gilliland in 1978) attempts to solve both the missing solar neutrino flux problem and the *missing mass* (dark matter) of the universe problem (Faulkner and Gilliland, 1985; Press and Spergel, 1985). They conjecture that WIMPS (weakly interacting massive particles; also called cosmions) created during the birth of the universe collect gravitationally around massive objects like stars and in particular our sun. In addition to accounting for the *missing mass*, WIMPS might reduce the core temperature of the sun and hence fusion rates to levels that account for the reduced neutrino flux. However, if dark matter has accumulated in the sun to the level that it enhances the sun's thermal conductivity enough to yield the measured solar luminosity with a lower central temperature than in the SSM, this dark matter could be directly detected by bolometric procedures, as pointed out by De Rujula *et al.* (1986). Needless to say, this dark matter has not yet been directly detected.

Even more fascinating hypotheses abound. One considers that a black hole has fallen into the center of the sun and is gobbling up the neutrinos. If it does not evaporate away first by Hawking radiation, it may eventually cause the sun to collapse. Another hypothesis considers the possibility that neutrinos may tunnel out of our universe. The existence of differing mass neutrinos makes this a testable hypothesis. In situations where all three kinds of neutrinos are present in known amounts, the relative percentages of the heavier neutrinos should increase with time, as the tunneling rate for a given energy will be the highest for the lightest one, the ν_e (Rabinowitz, 1990). A more bleak hypothesis is that the sun's core has burned out or is otherwise turned off, and it is only a question of time before the outer region of the sun cools off and affects life on the earth.

To our knowledge, we are the first to seriously question the predictions of the SSM, based upon the ramifications of a more realistic *S*-factor (Kim *et al.*, 1992). A very small minority (Morrison, 1992) has suggested that there are major discrepancies in the experimental data used as input for the SSM. This does not appear to be justified, and we accept the experimental data. As we shall show, it is the theoretical implementation of the experimental input data which has been erroneous. Johnson *et al.* (1992) improved upon the traditional Debye approximation for plasma screening. However, their analysis found corrections of $<2\%$ for the electron capture rate, and only about 7% for the solar proton capture rate, leading them to conclude that their approach reduces the ^{8}B neutrino flux by only about 7% from the currently accepted value.

Even if attempted solutions such as neutrino family oscillations and neutrino magnetic moment with change of chirality were capable of resolving the solar neutrino problem, a secondary problem would still remain unsolved. The standard solar model predicts that the sun's luminosity has increased by 40% over the last 5 billion years. This appears to be inconsistent with the earth's geological record, and is a cause of much concern to geophysicists and paleoclimatologists. An increase in the sun's luminosity should be accompanied by an exponential growth in the ^{8}B neutrino flux with a doubling time of 8.5×10^{8} years (Haxton, 1990). Our solution to the missing solar neutrino flux problem may be in the same category as all the other proposed solutions, and may also not resolve the paleoclimatological problem. The resolution of this associated problem may require a modification of the SSM.

2. RECENT RESULTS OF SOLAR NEUTRINO EXPERIMENTS

The processes $p + {}^{7}\mathrm{Be} \to {}^{8}\mathrm{B} + \gamma$ and ${}^{8}\mathrm{B} \to {}^{8}\mathrm{Be}^{*} + e^{+} + \nu$ (<15 MeV) produce neutrinos to which the ^{37}Cl detector (Davis, 1986, 1988) at Homestake Mine is sensitive. The average total rates of solar neutrino (electron type, ν) interactions $R_{\nu}^{\mathrm{Cl}}(\mathrm{exp})$ have been measured there by means of the reaction $\nu + {}^{37}\mathrm{Cl} \to e^{-} + {}^{37}\mathrm{Ar}$ from 1970 to the present. The observed average values are R_{ν}^{Cl} ([year, month] 70.3–85) $= 2.1 \pm 0.3$ SNU (Davis, 1986), R_{ν}^{Cl} (70.3–88.3) $= 2.33 \pm 0.25$ SNU (Davis, 1988), and R_{ν}^{Cl} (86.8–88.3) $= 4.2 \pm 0.7$ SNU (Davis, 1988) for periods from March 1970 to the end of 1985, from March 1970 to March 1988, and from August 1986 to March 1988, respectively, where a solar neutrino unit (SNU) is 10^{-36} interactions per target atom per second (Bahcall, 1969). Many theoretical estimates of R_{ν} based on standard solar models (SSM) have been carried out and the latest theoretical values are $R_{\nu}^{\mathrm{Cl}}(\mathrm{BU}) = 7.9 \pm 2.6$ SNU (Bahcall and Ulrich, 1988), $R_{\nu}^{\mathrm{Cl}}(\mathrm{TCCD}) = 5.8 \pm 1.3$ SNU (Turck-Chièze *et al.*, 1988), and R_{ν}^{Cl} (BP) $= 8.0 \pm 3.0$ SNU (Bahcall and Pinsonneault, 1992). The discrepancy between the theoretical values and the experimental data is the solar neutrino problem (or puzzle). In the theoretical estimates $R_{\nu}^{\mathrm{Cl}}(\mathrm{BU})$ and $R_{\nu}^{\mathrm{Cl}}(\mathrm{TCCD})$, the major contribution comes from $^8B \to {}^8Be^* + e^+ + \nu$, e.g., $R_\nu^{Cl}(BU, {}^8B) = 6.1 \pm 2.0$ SNU and $R_\nu^{Cl}(TCCD, {}^8B) = 4.0 \pm 0.9$ SNU. Most recently, a real-time, directional solar-neutrino signal has been observed in the water Cherenkov detector Kamikande-II (KAM-II) (Hirata *et al.*, 1989, 1990). The reported value of the neutrino flux is 0.46 ± 0.05 (stat.) ± 0.06 (syst.) relative to the standard solar model estimate of $R_\nu^{Cl}(SSM, 1988) = 7.9 \pm 2.6$ SNU. Since the KAM-II detector is sensitive mostly to

$$^8B \to {}^8Be + e^+ + \nu(<15\ \text{MeV})$$

we can interpret their observed solar neutrino flux as corresponding to a Cl equivalent rate of $R_\nu^{Kam}({}^8B) = 3.6 \pm 0.6$ SNU, which is consistent with the ^{37}Cl detector result of $R_\nu^{Cl}({}^8B) = 3.2 \pm 0.5$ SNU [$\approx 77\%$ $R_\nu^{Cl}(86.8$–$88.3)$] and with the calculated value of $R_\nu^{Cl}(TCCD, {}^8B) = 4.0 \pm 0.9$ SNU (Turck-Chièze *et al.*, 1988), but is inconsistent with the calculated value of $R_\nu^{Cl}(BU, {}^8B) = 6.1 \pm 2.0$ SNU (Bahcall and Ulrich, 1988).

The recent gallium measurement of the rate of production of ^{71}Ge from ^{71}Ga by solar neutrinos yields $R_\nu^{Ga}(\text{exp}) = 83 \pm 19$ (stat.) ± 8 (syst.) SNU (1 sigma) (Anselmann *et al.*, 1992), compared with SSM predictions of 125 ± 5 SNU (Turck-Chièze *et al.*, 1988) and $131.5 \pm 21/17$ SNU (Bahcall and Pinsonneault, 1992), of which major contributions of 70.8, 35.8, and 13.8 SNU are from reactions $p(p, e^+\nu)D$, $^7Be(e^-, \nu)^7Li$, and $^7Be(p, \gamma)^8B(e^+\nu)^8Be^*(\alpha)^4He$, respectively.

3. NUCLEAR PHYSICS INPUT

The solar neutrino flux is calculated using low-energy nuclear fusion cross sections $\sigma(E)$ as input data. Since $\sigma(E)$ at solar energies ($\lesssim 20$ keV) cannot be measured in the laboratory, they are extracted from the laboratory measurements of $\sigma(E)$ at higher energies by an extrapolation procedure based on nuclear theory. However, the energy dependence of the nuclear reaction cross section $\sigma(E)$ cannot be obtained rigorously from first principles, since the many-nucleon scattering problem cannot be solved exactly even if the nucleon–nucleon force is given. Therefore, one must rely on physically reasonable nuclear reaction models, such as optical potential models (OPM) (Barker, 1980; Kim *et al.*, 1987) and resonating group models (RGM) (Kolbe *et al.*, 1988; Johnson *et al.*, 1992). Both OPM and RGM have been used to extract low-energy $\sigma(E)$ for $^7Be(p, \gamma)^8B$. Reliability of the predicted energy dependence of $\sigma(E)$ for $^7Be(p, \gamma)^8B$ obtained from OPM and RGM calculations (Barker, 1980; Kim *et al.*, 1987; Kolbe *et al.*, 1988; Johnson *et al.*, 1992) cannot be established due to the model assumptions made and the many parameters used and also due to the neglect of meson exchange currents and the Pauli exclusion principle in these calculations (Barker, 1980; Kim *et al.*, 1987; Kolbe *et al.*, 1988; Johnson *et al.*, 1992). Values of the astrophysical S-factor extracted from OPM calculations vary by a factor of ~2 (0.014–0.027 keV-b) due to uncertainties in OPM parameters used (Barker, 1980). Even for the $E1$ operator ($\boldsymbol{\varepsilon} \cdot \mathbf{r}$) used for ^{7}Be($p$, γ)^{8}B in the long-wavelength limit ($E_\gamma \to 0$), gauge invariance has not been imposed, since the initial (^{7}Be$+p$) and final ($\gamma + {}^8$B) states in these model calculations (Barker, 1980; Kim *et al.*, 1987; Kolbe *et al.*, 1988; Johnson *et al.*, 1992) are not exact eigenstates of the same nuclear Hamiltonian for the eight-nucleon system. Hence, Siegert's theorem (Siegert, 1937; Austern and Sach, 1951; Foldy, 1953) cannot be used to justify the use of the $E1$ ($\boldsymbol{\varepsilon} \cdot \mathbf{r}$) operator excluding the meson exchange current contributions in these OPM and RGM calculations. Furthermore, these models use an unrealistic point-charge approximation for the $E1$ operator (Foldy, 1953). Therefore, we use the least model-dependent parametrization procedure based on a barrier transformission model (BTM). Such a procedure has been used extensively in astrophysical problems (Fowler *et al.*, 1967) involving the Gamow transmission coefficient for the Coulomb barrier (Gamow, 1928; Blatt and Weisskopf, 1952). However, we use an improved and more realistic barrier transmission solution for extrapolating $\sigma(E)$ to lower energies.

Previous low-energy (<20 keV) $\sigma(E)$ used in the standard solar model calculations (Bahcall and Ulrich, 1988; Turck-Chièze *et al.*, 1988) are calculated by extrapolating the experimental values of $\sigma(E)$ at higher energies using the parametrization (Fowler *et al.*, 1967):

$$\sigma(E) = \frac{S(E)}{E} \exp\left[-\left(\frac{E_G}{E}\right)^{1/2} \right] \tag{1}$$

where $E_G = (2\pi \alpha Z_1 Z_2)^2 \mu c^2/2$ with the reduced mass $\mu = m_1 m_2/(m_1 + m_2)$ and E is the center-of-mass (CM) kinetic energy. The transmission coefficient ("Gamow" factor) $\exp[-(E_G/E)^{1/2}$ results from the approximation $E \ll B$ (Coulomb barrier height), representing the probability of bringing two charged particles to zero separation distance. This implies that the Coulomb barrier $Z_1 Z_2 e^2/r$ also exists inside the nuclear surface of radius R, which is unphysical and unrealistic. In order to accommodate more realistic transmission coefficients, we write a more general parametrization for $\sigma(E)$ as

$$\sigma(E) = \frac{\tilde{S}(E)}{E} T(E) \tag{2}$$

where $T(E)$ is the proton transmission coefficient as derived by Blatt and Weisskopf (1952) in a generalized form. To our knowledge, we are the first to apply it to this problem. In the following section, we describe several alternative models for the transmission coefficient $T(E)$.

4. TRANSMISSION COEFFICIENT

4.1. Coulomb Barrier with Nuclear Square-Well Potential

If the target nucleus ^7Be is assumed to have an interior square-well nuclear potential and an exterior Coulomb repulsive potential,

$$V(r)=\begin{cases}-V_0 & r<R \\ Z_1Z_2e^2/r, & r\geq R\end{cases} \tag{3}$$

then the proton transmission coefficient is given by

$$T(E)=\frac{4s_0KR}{\Delta_0^2+(s_0+KR)^2} \tag{4}$$

where

$$s_0=R[(G_0F_0'-F_0G_0')/(G_0^2+F_0^2)]_{r=R} \tag{5}$$

$$\Delta_0=R[(G_0G_0'+F_0F_0')/(G_0^2+F_0^2)]_{r=R} \tag{6}$$

and $\hbar^2K^2/2\mu=V_0+E$, with $E=\hbar^2k^2/2\mu$. Here F_0' and G_0' are derivatives of F_0 and G_0 with respect to r, where F_0 and G_0 are the regular and irregular Coulomb wave functions normalized asymptotically ($r\to\infty$) as

$$F_0(r)\approx\sin[kr-\eta\ \ln(2kr)+\sigma_0]$$

$$G_0(r)\approx\cos[kr-\eta\ \ln(2kr)+\sigma_0]$$

σ_0 is the S-wave Coulomb phase shift, $\sigma_0=\arg\Gamma(1+i\eta)$ with the Sommerfeld parameter $\eta=Z_1Z_2e^2/\hbar v$. The incoming flux is normalized to unity, $G_0^2+F_0^2=1$ for $r\to\infty$.

Our calculation of the proton transmission coefficient $T(E)$ is done with $V_0=46$ MeV and $R=3.2$ fm [(1.25 fm)$A^{1/3}+0.8$ fm (proton radius)], using equations (5) and (6). In Figure 1, the values of $T(E)$ calculated using equation (4) are plotted (solid curve) and compared with the traditional "Gamow" form $\exp[-(E_G/E)^{1/2}]$ used in equation (1) (dashed curve).

4.2. Approximate WKB Solution for $T(E)$

For the potential barrier given by equation (3), an approximate solution for $T(E)$ can be calculated in the Wentzel–Kramers–Brillouin (WKB) approximation as

$$T_R^{\mathrm{WKB}}(E)=\exp\left\{-2\left(\frac{2\mu}{\hbar^2}\right)^{1/2}\int_R^{r_a}\left(\frac{Z_1Z_2e^2}{r}-E\right)^{1/2}dr\right\}$$
$$=\exp\left(-\left(\frac{E_G}{E}\right)^{1/2}\left(\frac{2}{\pi}\right)\left\{\cos^{-1}\left[\left(\frac{E}{B}\right)^{1/2}\right]-\left(\frac{E}{B}\right)^{1/2}\left(1-\frac{E}{B}\right)^{1/2}\right\}\right) \quad (7)$$

where $B=Z_1Z_2e^2/R$ and r_a is the classical turning point, $Z_1Z_2e^2/r_a=E$. Note that $T_R^{\mathrm{WKB}}(E)$ is defined only for $E\leq Z_1Z_2e^2/R$ (Coulomb barrier height) and $T_R^{\mathrm{WKB}}(Z_1Z_2e^2/R)=1$, whereas the exact solution, equation (4), is valid for all values of E. Using equation (7), we plot the calculated values of $T_R^{\mathrm{WKB}}(E)$ with $R=3.2$ fm ($B=1.800$ MeV) (dotted curve) and compare them with those of $T(E)$, equation (4), (solid curve) in Figure 1.

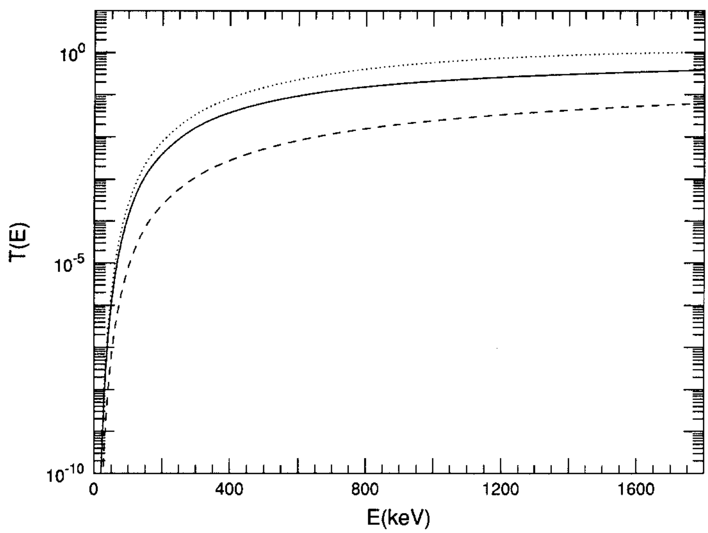

Fig. 1. The traditional transmission coefficient for $^7Be(p,\gamma)^8B$, $T_{R=0}^{\mathrm{WKB}}(E)$ (dashed curve), compared to that obtained in the WKB approximation with $R=3.2$ fm (dotted curve) and that obtained assuming an interior square-well and exterior Coulomb repulsive potential, equation (4), with $R=3.2$ fm and $V_0=46$ MeV (solid curve).

4.3. Traditional Transmission Coefficient

The traditional transmission coefficient used in equation (1) can be obtained from equation (7) with $R=0$ (or equivalently $E \ll B$):

$$T_{R=0}^{\mathrm{WKB}} = \exp\left\{-2\left(\frac{2\mu}{\hbar^2}\right)^{1/2} \int_0^{r_a} \left(\frac{Z_1 Z_2 e^2}{r} - E\right)^{1/2} dr\right\}$$

$$= \exp\left[-\left(\frac{E_G}{E}\right)^{1/2}\right] \tag{8}$$

The values of $T_{R=0}^{\mathrm{WKB}}(E)$ obtained from equation (8) (dashed curve) are compared with $T(E)$ (solid curve) and $T_{3.2\,\mathrm{fm}}^{\mathrm{WKB}}(E)$ (dotted curve) in Figure 1.

5. REVISED ASTROPHYSICAL *S*-FUNCTION

We calculate the proton transmission coefficient $T(E)$ using equation (4) with the depth and width of the square well $V_0 = 46$ MeV and $R = (1.25\text{ fm})A^{1/3} + 0.8$ fm (proton radius) $= 3.2$ fm, respectively. A new (experimental) S-factor (shown in Figure 1), $\tilde{S}_{\mathrm{exp}}(E)$, is derived using the expression

$$\tilde{S}(E) = \frac{\sigma_{\mathrm{exp}}(E)E}{T(E)} \tag{9}$$

obtained from equation (2), and the experimental cross-section data $\sigma_{\mathrm{exp}}(E)$ (Kavanagh *et al.*, 1969; Filippone *et al.*, 1983).

The (theoretical) S-factor was then parametrized as

$$\tilde{S}(E) = \tilde{S}_{\mathrm{NR}}(E) + \tilde{S}_{\mathrm{R}}(E) = \sum_{i=0}^{n} S_i E^i + \frac{G}{(E_r - E)^2 + \Gamma^2/4} \tag{10}$$

where $\tilde{S}_{\mathrm{NR}}(E)$ and $\tilde{S}_{\mathrm{R}}(E)$ are the nonresonant and resonant ($E_r \approx 630$ keV) contributions, respectively. The convergence of the extracted value of $\tilde{S}_{\mathrm{NR}}(0) = S_0$ as a function of the number $(n+1)$ of terms in the Maclaurin series $\tilde{S}(E) = \tilde{S}_{\mathrm{NR}}(E) + \tilde{S}_{\mathrm{R}}(E) = \sum_{i=0}^{n} S_i E^i$ may depend on the amount and quality of the data available. We adopt a quadratic Maclaurin series ($n=2$) for $\tilde{S}_{\mathrm{NR}}(E)$ as is customarily done in the conventional parametrization of $S(E)$, equation (1) (Fowler *et al.*, 1967). The parameters S_i, G, E_r, and Γ were then found via a χ^2 fit to the "experimental" S-factor, $S_{\mathrm{exp}}(E)$, with the results $S_0 = 0.0818 \times 10^{-3}$ keV-b, $S_1 = 0.940 \times 10^{-6}$ b, $S_2 = 6.55 \times 10^{-10}$ keV^{-1}-b, $E_r = 632.4$ keV, $\Gamma = 38.16$ keV, and $G = 2.51$ keV^3-b, for the $\sigma_{\mathrm{exp}}(E)$ data of Filippone *et al.* (1983). For the $\sigma_{\mathrm{exp}}(E)$ of Kavanagh *et al.* (1969), we obtain $E_r = 631.4$ keV, $\Gamma = 39.26$ keV, $G = 3.30$ keV^3-b, $S_0 = 1.080 \times 10^{-3}$ keV-b, $S_1 = 1.300 \times 10^{-6}$ b, and $S_2 = 1.030 \times 10^{-9}$ keV^{-1}-b.

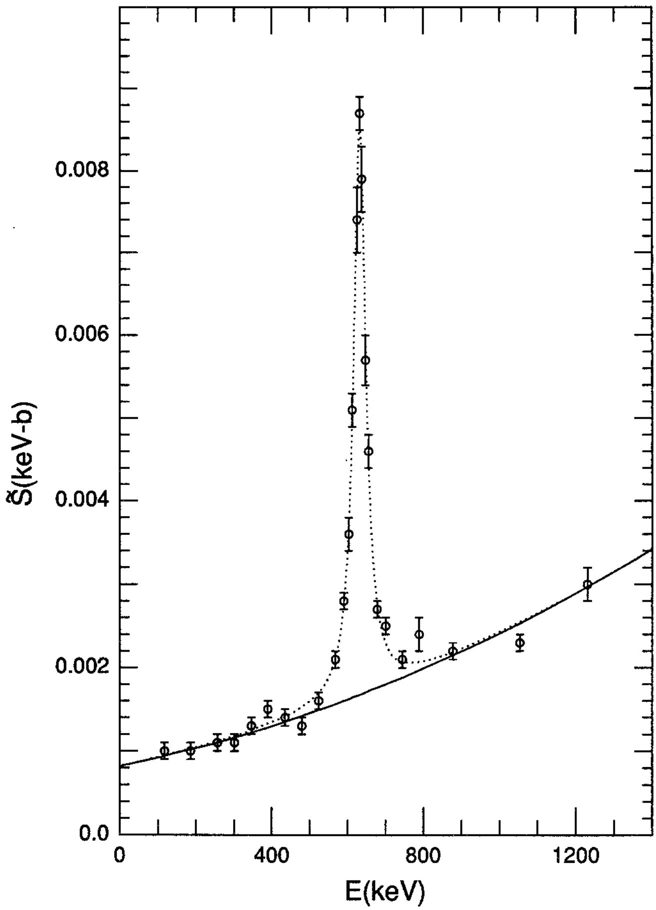

Fig. 2. "Theoretical" S-factor $\tilde{S}(E)$, equation (10), compared to the "experimental" $\tilde{S}_{exp}(E)$ resulting from the cross-section data of Filippone *et al.* (1983).

These values of the parameters are used to calculate $\tilde{S}(E)$, equation (10), and the calculated results for "theoretical" $\tilde{S}(E)$ are plotted along with the "experimental" S-factor, $\tilde{S}_{exp}(E)$, in Figures 2 and 3, for the Filippone *et al.* (1983) data (circles) and for the Kavanagh *et al.* (1983) data (diamonds), respectively.

6. NEW EXTRAPOLATED CROSS SECTION AND SOLAR NEUTRINO FLUX

The new resultant $\tilde{S}(E)$, equation (10), is in turn used to calculate $\sigma_{new}(E)$ at lower energies $E < 20$ keV using equation (2). The calculated

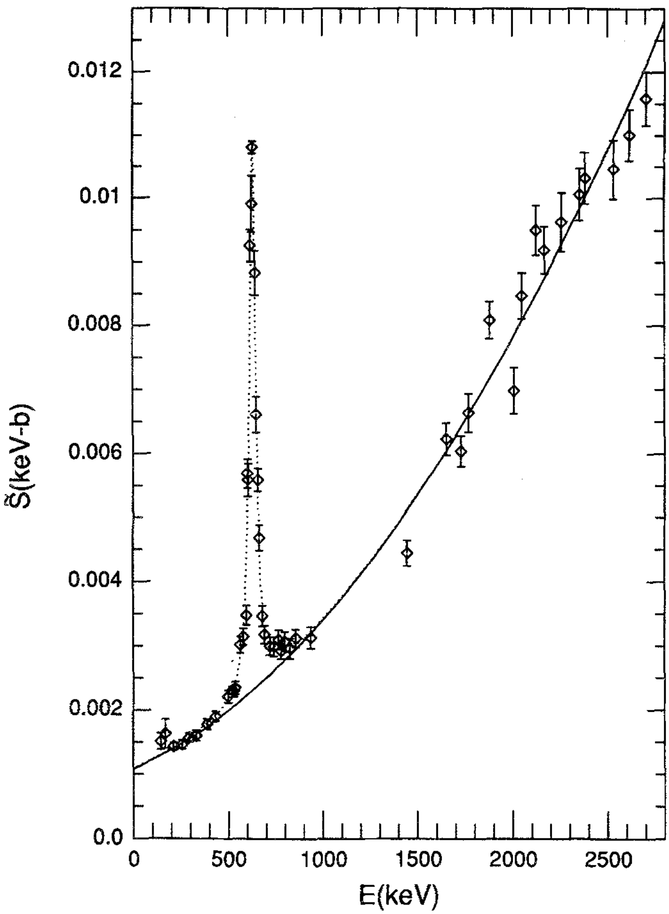

Fig. 3. Same as Figure 2, but for the data of Kavanagh *et al.* (1969).

values of $\sigma_{new}(E)$ are shown in Figure 4 for the data of Filippone *et al.* (1983) (circles) and in Figure 5 for the data of Kavanagh *et al.* (1969) (diamonds). They are compared in Table I with the old values of $\sigma_{BU}(E) = (0.0243 \text{ keV-b}) \{\exp[-(E_G/E)^{1/2}]\}/E$ (Bahcall and Ulrich, 1988) and $\sigma_{TCCD}(E) = (0.021 \text{ keV-b}) \{\exp[-(E_G/E)^{1/2}]\}/E$ (Turck-Chièze *et al.*, 1988), which were used in the previous SSM calculations of the solar neutrino rate, where the low-energy S-factors were obtained from OPM (Barker, 1980). Our extrapolation method involves two parameters $V_0 \approx 46$ MeV and R, but our results for $\sigma(E)$ at low energies are insensitive to V_0, since V_0 is effectively determined and absorbed in $\tilde{S}(E)$ when $\tilde{S}(E)$ is fitted to the experimental data. Our extrapolated $\sigma(E)$ vary only by ~2% when R is

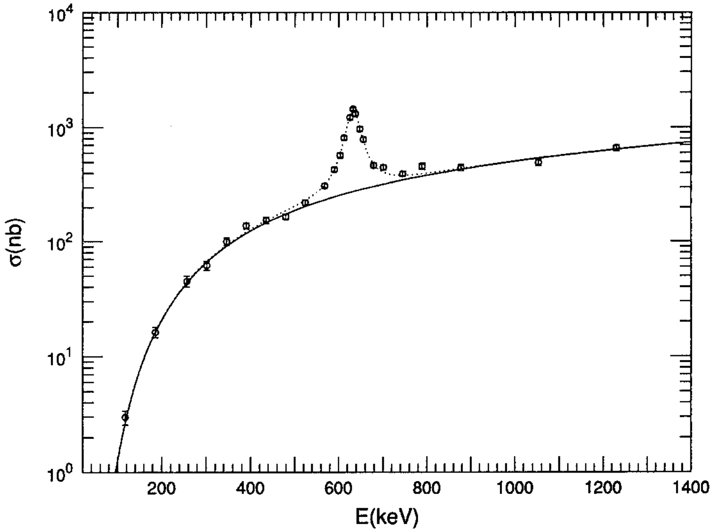

Fig. 4. Cross-section $\sigma_{new}(E)$ resulting from the "theoretical" S-factor plotted in Figure 2 along with the data of Filippone *et al.* (1983).

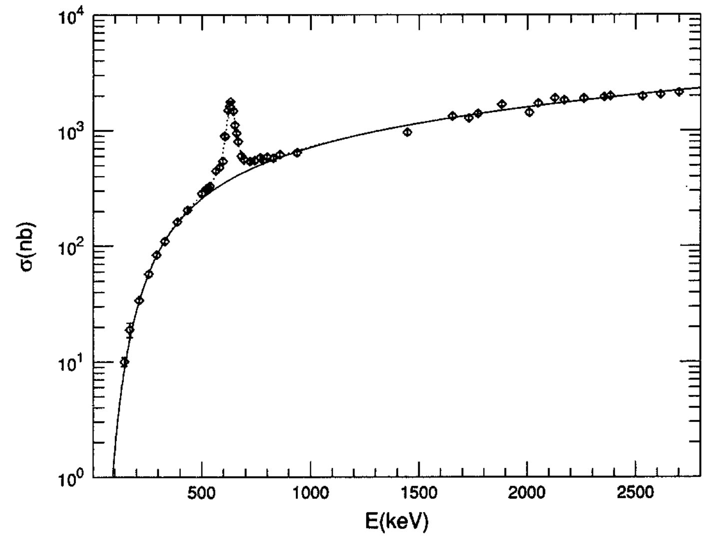

Fig. 5. Cross-section $\sigma_{new}(E)$ resulting from the "theoretical" S-factor plotted in Figure 3 along with the data of Kavanagh *et al.* (1969).

Table I. Comparison of the $^7Be(p, \gamma)^8B$ Cross Section σ_{new} Calculated Using Equations (2) and (10) with Previous Calculations σ^{BU} (Bahcall and Ulrich, 1988) and σ_{TCCD} (Turck-Chièze *et al.*, 1988) Along with the Resulting Solar Neutrino Rates $\phi\sigma(^8B)_{BU}$ and $\phi\sigma(^8B)_{TCCD}$[a]

E(CM) (keV)	σ_{new} (nb)	σ_{new}/σ_{BU}	$\phi\sigma(^8B)_{BU}$ (SNU)	$\sigma_{new}/\sigma_{TCCD}$	$\phi\sigma(^8B)_{TCCD}$ (SNU)
1	1.445×10^{-44}	0.633	3.86	0.732	2.93
	(1.908×10^{-44})	(0.836)	(5.10)	(0.966)	(3.87)
5	4.668×10^{-17}	0.635	3.87	0.735	2.94
	(6.164×10^{-17})	(0.838)	(5.11)	(0.970)	(3.88)
10	1.130×10^{-10}	0.637	3.89	0.737	2.95
	(1.493×10^{-10})	(0.842)	(5.13)	(0.974)	(3.89)
20	3.024×10^{-6}	0.640	3.90	0.741	2.96
	(3.997×10^{-6})	(0.846)	(5.16)	(0.979)	(3.92)

[a]The top values in each row result from a fit to the experimental cross-section data of Filippone *et al.* (1983), while the bottom values (in parentheses) result from the data of Kavanagh *et al.* (1969).

changed from $R=(1.25\text{ fm})A^{1/3}=2.39\text{ fm}$ to $R=3.2\text{ fm}$. The extracted resonance parameters E_r, Γ, and G in equation (10) are also insensitive to variations of $T(E)$ due to the above changes in R.

In a similar manner, we fit the same experimental cross-section data using equation (1) and a form for $S(E)$ analogous to equation (10). The calculated values of $S(E)$ are plotted in Figures 6 and 7 for the data of Filippone *et al.* (1983) (circles) and Kavanagh *et al.* (1969) (diamonds), respectively. Our extrapolated values of low-energy (<20 keV) $\sigma(E)$ using equation (2) are lower (by $\lesssim 10\%$) than those obtained using equation (1), which indicates that both equations (1) and (2) can provide a reasonable model-independent procedure for extrapolating $\sigma_{exp}(E>100\text{ keV})$ to lower energies, in contrast to the model uncertainty involved in OPM calculations.

Table I shows how well the results of our analysis agree with the neutrino detection measurements, using new and older experimental input data. Our closest agreement comes from our calculated value of 3.9 SNU using the newer experimental input data (Filippone *et al.*, 1983), which agrees quite well with the Kamikande detector result of 3.6 ± 0.7 SNU.

In the interest of thoroughness we point out that if we use older high-energy cross-sectional data as input to our calculations, our results are not as impressive in comparison with the SSM values. However, even these results are still generally significant, and we summarize them here. If we use the Kavanagh *et al.* (1969) experimental results, $\sigma_{exp}(E)$, as input data, our calculated $\sigma_{new}(E)$ for $E\leq 20$ keV are not quite as small as previously. In this case (comparing as before) for (^8B) we obtain 5.1 SNU rather than 6.1 SNU, and 3.9 SNU rather than 4.0 SNU.

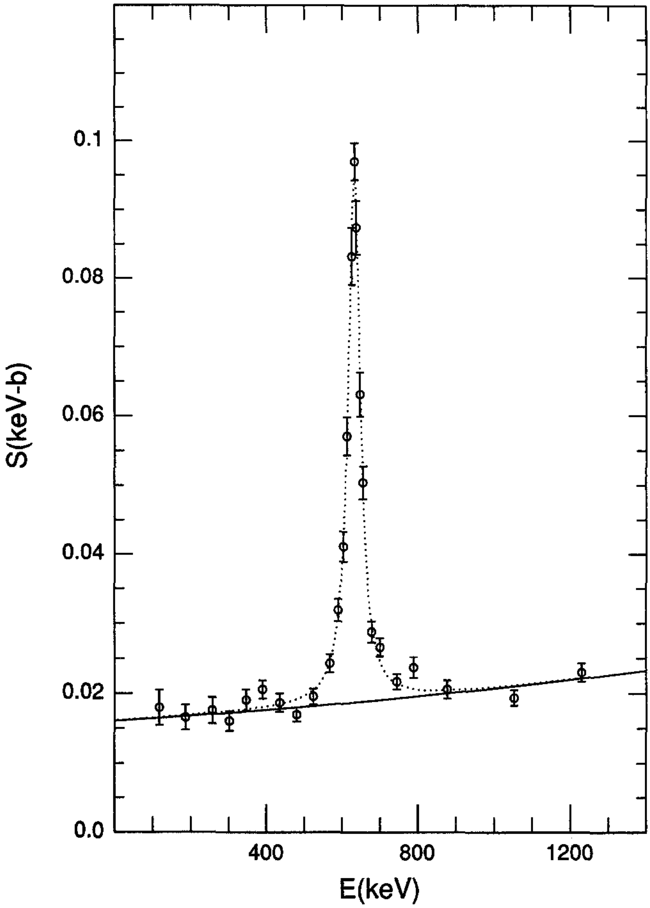

Fig. 6. Fit to the S-factor resulting from the cross-section data of Filippone *et al.* (1983) and equation (1). The dotted line shows the full S-factor, $S_R(E)+S_{NR}(E)$, while the solid line is the nonresonant S-factor, $S_{NR}(E)$.

7. CONCLUSIONS

Although low-energy $\sigma(E)$ extracted from $S(E)$, equation (1), and $\tilde{S}(E)$, equation (2), are very similar within $\lesssim 10\%$ for the $^7Be(p, \gamma)^8B$ reaction, they may be substantially different for other reactions which have different barrier heights. Furthermore, an important advantage of using equation (2) with equation (4) for proton tunneling is that equations (2) and (4) for the (p, γ) reaction are directly related to the (n, γ) reaction. For the (n, γ) reaction, the replacement of F_0 and G_0 in equations (4)–(6), with the spherical Bessel functions j_0 and n_0, leads to the appropriate energy

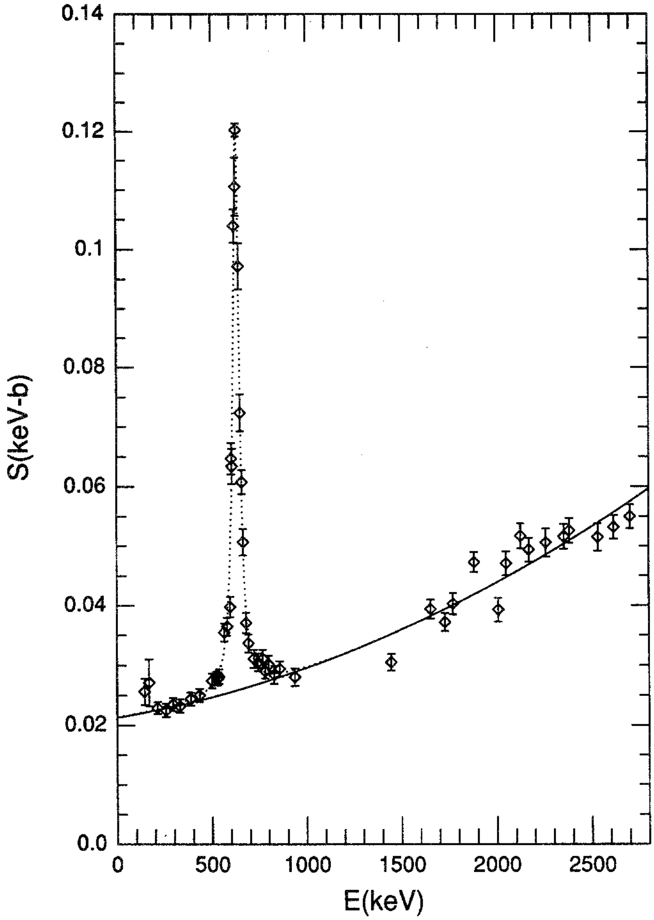

Fig. 7. Same as Figure 6, but for the data of Kavanagh *et al.* (1969).

dependence for the (n, γ) cross section, $\sigma(E) \propto \tilde{S}(E)/v$, at low energies (Blatt and Weisskopf, 1952). However, the use of the conventional form, equation (1), cannot provide the direct relationship between (p, γ) and (n, γ) reactions. It is possible to test the validity of the extrapolation procedure for (p, γ) reactions using the thermal neutron capture cross section for (n, γ). A similar comparison was previously attempted for the case of $D(p, \gamma)^3He$ and $D(n, \gamma)T$ reactions (Griffiths *et al.*, 1963).

A further reduction of our theoretical value of the solar neutrino interaction rate $[\phi\sigma(^8B)]$ may result if other fusion cross sections, e.g., $^3He(\alpha, \gamma)^7Be$, are also found to be decreased. Therefore, reinvestigation of extrapolation procedures for other fusion reaction cross sections in the p–p

chain are also desirable, and may bring our theoretical results into even closer agreement with the experimental results. Since the solar luminosity is mainly linked with energy release from p–p fusion, the lower neutrino flux and concomitantly lower fusion rates resulting from lower cross sections further up in the fusion chain hardly affect the sun's luminosity.

We have succeeded in showing that our conventional nuclear physics solution to the solar neutrino problem comes closer to solving this long-standing problem than all previous approaches, including some rather provocative ones. Our calculated values of $\sigma_{new}(E)$ for $E \leq 20$ keV derived from the experimental input data $\sigma_{exp}(E)$ (Filippone *et al.*, 1983) are ~36% smaller than the commonly accepted values of Bahcall and Ulrich (1988), $\sigma_{BU}(E)$. Our results are 26% lower than the values of Turck-Chièze *et al.* (1988). The comparisons are summarized in Table I. Our analysis thus reduces the presently presumed SSM values for (^{8}B) of 6.1 SNU (Bahcall and Ulrich, 1988) to 3.9 SNU; and 4.0 SNU (Turck-Chièze *et al.*, 1988) to 3.0 SNU. Thus our (^{8}B) results are more consistent with the Homestake Mine Cl detector results of 2.1 ± 0.3 SNU ([year, month] = 70.3–85), 2.33 ± 0.25 SNU (70.0–88.3), and 4.2 ± 0.7 SNU (86.8–88.3), and also the the Kamikande detector result of 3.6 ± 0.7 SNU. In all the above cases, including additional reactions may further reduce the existing discrepancy between theory and experiment. Our solution does not change the expected spectrum of ^{7}Be(p, γ)^{8}B neutrinos. It only reduces their flux. Our solution does not challenge the standard solar model, but rather it challenges theoretically derived cross sections that are used as input for the SSM.

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